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Level 7

Focused Mathematics Intervention

Teacher's Guide

Teacher Created Materials

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Kit Components

HOW TO USE This product

Teacher's Guide

30 easy-to-use, standards-based lesson plans



Student Guided Practice Book

Full-color student activities



Assessment Guide

Includes a pretest, posttest, performance tasks with assessments, and the answer key for the *Student Guided Practice Book*



3 Math Fluency Game Sets

Include a game board, directions, an answer key, and game pieces



3 Digital Math Fluency Games

Focus on mathematical skills and strategies, and are on the Digital Resources USB Device



Digital Resources

- PDFs of all student materials, game sets, activity sheets, assessments, etc.
- PDFs of teacher resources
- Digital Math Fluency Games
- Electronic versions of the Pretest, Posttest, Performance Tasks, and reporting tools

Refocus Mini Lesson



Provide as PowerPoint[®] and PDF files



Teaching a Lesson

Teacher's Guide

Each 8-page lesson is organized in a consistent format for ease of use. Teachers may choose to complete some or all of the lesson activities to best meet the needs of their students. Lesson materials can be utilized flexibly in a variety of settings. For example, modeling with a small group, using printed materials with a document camera, or using PDF materials on a digital platform, such as an interactive whiteboard. Each lesson includes:

- an overview page with key information for planning
- key mathematics content standards covered
- key mathematical practices and processes addressed
- an overview providing teacher background or student misconceptions
- a Warm-Up activity to build students' recall of important mathematical concepts
- a whole-class Language and Vocabulary activity
- time markers to indicate the approximate time for instruction
- a whole-class section focusing on the key concept/skill being taught
- use of the gradual release of responsibility model in the Whole-Group lesson section
- differentiation strategies to support and extend learning with the Refocus lesson and Extend Learning activity
- math fluency games that motivate students to develop and reinforce mastery of basic skills
- a Math in the Real World concept task activity









HOW TO USE THIS PRODUCT Teaching a Lesson (cont.)

Student Guided Practice Book

Each lesson in the *Teacher's Guide* has seven corresponding student pages in the *Student Guided Practice Book:*

- a We Do activity to support the gradual release of responsibility model
- a You Do activity to facilitate independent practice
- a Quick Check to easily monitor students' progress
- a Refocus activity for students who need more instruction
- an Independent Practice page to reinforce mathematical content taught in the lesson
- a Math in the Real World concept task for students to apply the math concept in a real-life scenario
- a Reflection page for students to share their mathematical understanding



Learning Objectives

Ratios and Proportional Relationships

• Identify the constant of proportionality (unit rate) in tables, graphs, and equations.

Mathematical Practices and Processes

- Reason abstractly and quantitatively.
- Attend to precision.
- Look for and make use of structure.

Progress Monitoring

The *Student Guided Practice Book* pages below can be used to formally and informally assess student understanding of the concepts.



Materials

• Student Guided Practice Book (pages 20–26)

LESSON

- Math Fluency Game Sets
- Digital Math Fluency Games
- different-colored highlighters

Teacher Background

In this lesson, students will use what they know about unit rates to identify the constant of proportionality in tables, graphs, and equations. The unit rate is found by dividing the numerator by the denominator of a ratio. The unit rate is the constant of proportionality. The constant of proportionality in an equation is the coefficient of x (independent variable). In a table or graph, the constant of proportionality is y, when x = 1. Students should be familiar with how points are graphed on the coordinate plane. A point with coordinates (x, y)is graphed by starting at the origin (0, 0), moving *x*-units right or left, and moving *y*-units up or down.

Warm-Up 10 min.

LESSON

1. Draw the following table on the board:

x	y
5	15
6	18
7	21
8	24

Say, "Find the unit rate using $k = \frac{y}{x}$ for each of the lines in the table." $(\frac{3}{t})$

- **2.** Remind students that to find the unit rate they must divide the value for y by the value for x.
- **3.** Some students may think the answer is $\frac{1}{3}$ instead of $\frac{3}{1}$. If that happens, remind students that they need to find the unit rate using $k = \frac{y}{x}$.
- **4.** Ask students if the table represents a proportional relationship. Students should find that the unit rate, or constant of proportionality, remains the same for each ordered pair. Therefore, the table represents a proportional relationship.

Language and Vocabulary 10 min.

1. At the beginning of the lesson, review the following vocabulary terms:

unit rate constant of proportionality proportional relationship coefficient independent variable dependent variable

- **2.** Say, "We discovered that the *unit rate* for the table in the Warm-Up is $\frac{3}{1}$. This is also called the *constant of proportionality* for the table. The constant of proportionality is represented by *k* in an equation for a *proportional relationship*.
- **3.** Point out that the *coefficient* will be multiplied by the *independent variable* to equal the *dependent variable* in an equation that represents a *proportional relationship*. Write the following on a large sheet of paper and have students copy it into their journals or a sheet of paper:

unit rate = constant of proportionality = k = coefficient of x

$$k = \frac{y}{r}$$

y = dependent variable

x = independent variable

dependent variable = (constant of proportionality)(independent variable)

y = kx

Whole-Group Lesson 40 min.

Focus

- The following lesson will address this focus question: How do you find the constant of proportionality for a table, a graph, and an equation?
- **2.** You may wish to write the focus question on the board and read it aloud to students. Explain that you will revisit the focus question at the end of the lesson.

I Do

- Say, "Today we are going to find the constant of proportionality in tables, graphs, and equations. During the Warm-Up, we reviewed finding the unit rate. The unit rate is also known as the constant of proportionality."
- **2.** Say, "The table represents Adam's walk with his dog on a Saturday afternoon." Write the following table on the board:

Time (hours)	Distance (miles)	distance/ time = speed
$\frac{1}{4}$	$\frac{3}{4}$	
$\frac{1}{2}$	$1\frac{1}{2}$	
$\frac{3}{4}$	$2\frac{1}{4}$	
$1\frac{1}{4}$	$3\frac{3}{4}$	
$1\frac{1}{2}$	$4\frac{1}{2}$	

3. Say, "For this table, we need to find the unit rate of distance per unit of time for each line of the table. So, distance/time = rate of speed. Remember we will be writing and simplifying complex fractions. Let's find the constant of proportionality for each ordered pair in the table."

LESSON

Whole-Group Lesson (cont.)

4. Ask, "What is the unit rate?" $(\frac{3}{1})$ Write the unit rate $\frac{3}{1}$ in the last column for each ordered pair. Say, "In each case, the unit rate is 3 miles per hour. When we want to write an equation for this relationship, this unit rate becomes the constant of proportionality. In this case, the independent variable represents the distance. The amount of time it takes to walk a distance depends on the amount of distance walked."

- **5.** Say, "The equation we need to use is based on the formula for speed." Write on the board $\frac{d}{t} = s$, so d = st. Say, "To write an equation that represents this table, we can write d = 3t. The distance depends on the rate or constant of proportionality multiplied by the time."
- 6. Display the following graph on the board:



- 7. Say, "Look at this graph. How do we know it is proportional?" (It passes through the origin and is a straight line.) Say, "We can write an equation to represent this line. The constant of proportionality is also the steepness or slope of the line. We find the constant of proportionality by finding the ratio of y to x." Ask students to tell you the points on the line as you write them on the board. ((4, 3), (8, 6), (12, 9), (16, 12)) Say, "The rate is the ratio of y to x. To find the rate or constant of proportionality, we write fractions and simplify them." Have students determine the constant of proportionality for the ordered pairs.
- **8.** Say, "The constant of proportionality is $\frac{3}{4}$. This means that the value for *y* is always $\frac{3}{4}$ times *x* for this graph." As you say this, write $y = \frac{3}{4}x$ on the board.
- **9.** Write the equation y = 7x on the board. Ask, "Based on the work we have done so far, what is the constant of proportionality for this equation?" (7) Ask, "What does this mean?" (It means that y is 7 times x; or in a graph, every time you move 1 unit along the x-axis, you move 7 units along the y-axis.)

LESSON

I Do

(cont.)

Whole-Group Lesson (cont.)

We Do

- 1. Refer students to the Constantly Proportional activity sheet (*Student Guided Practice Book*, page 20). Say, "Let's look at more examples of tables and graphs together. First, look at the table in Question 1."
- 2. Ask, "How do you find the constant of proportionality for this table?" (Divide the number of apples by the number of pies.) Have students work with a partner to find the constant of proportionality and complete the table. Then, have students share their answers and explain their thinking with the class. Students should have found that the constant of proportionality is ⁵/₁, or 5.
- **3.** Look at Question 2. Ask, "How do you find the constant of proportionality for this graph?" (*Find points along the graph and divide the value of* y *by the value of* x.) Then, have students share their answers and explain their thinking with the class. Students should have found that the constant of proportionality is $\frac{5}{1}$, or 5.
- **4.** Look at Question 3. Ask, "How do we find the constant of proportionality in an equation?" (*We look for the coefficient of the independent variable.*) Have students do a choral response by saying the value of the constant of proportionality on the count of three. Confirm with students that the solutions are 8 and $\frac{1}{2}$.
- **5.** Have students explain how they found the constant of proportionality. To help students explain their reasoning, provide them with the following sentence frames:
 - In a table, I can find the constant of proportionality by _____ the value of the _____ variable by the _____ variable. (dividing; dependent; independent)
 - In a graph, I can find the constant of proportionality by _____ the value of _____ by the value of _____. (dividing; y; x)
 - In an equation, the constant of proportionality is _____ in the equation y = kx, and is the _____ of the independent variable. (k; coefficient)

Language Support

Each time you use the terminology *constant of proportionality*, *coefficient, independent variable*, and *dependent variable*, point to them in the problem. This will enable students to make connections by associating a vocabulary term to a mathematical symbol in an equation.

LESSON

Whole-Group Lesson (cont.)

You Do

LESSON

- 1. Refer students to the Proportionally Constant activity sheet (*Student Guided Practice Book*, page 21). Encourage students to refer to the process to find the constant of proportionality and identify the dependent and independent variables for tables, graphs, and equations.
- **2.** Have students share their solutions and reasoning. If students have difficulty explaining their reasoning, remind them to use the sentence frames and vocabulary terms.

Closing the Whole-Group Lesson

Revisit the focus question: *How do you find the constant of proportionality for a table, a graph, and an equation?* Ask students to explain how to find the constant of proportionality in tables, graphs, and equations. Students should explain that in a table or graph, the constant of proportionality can be found by dividing the value of y (dependent variable) by the value of x (independent variable). In an equation, the constant of the proportionality is the coefficient of x (independent variable).

Progress Monitoring (5) min.

- 1. Have students complete the Quick Check activity sheet (*Student Guided Practice Book*, page 22) to gauge student progress toward mastery of the Learning Objectives.
- 2. Based on the results of the Quick Check activity sheet and your observations during the lesson, identify students who may benefit from additional instruction in the Learning Objectives. These students will be placed into a small group for reteaching. See instructions on the following page.

Differentiated Instruction 🙆 min.

Gather students for reteaching. The remaining students will complete the Independent Practice activity sheet (*Student Guided Practice Book*, page 24) to reinforce their learning and then play the Math Fluency Games.

Refocus

Revisit the focus question for the lesson: *How do you find the constant of proportionality for a table, a graph, and an equation?* For students who are struggling to find the constant of proportionality, review how to find the independent and dependent variables. Have students highlight the dependent variable in one color, and the independent variable in another color. Remind students that the dependent variable depends on the independent variable. For example, the number of highlighters needed for the class depends on the number of students. Ask students to determine the dependent variable (*number of highlighters*) and the independent variable (*number of students*). Have students look at Question 1. Ask them to identify the independent and dependent variables and highlight them in different colors. Finally, support students as they complete Question 1 on the Refocus activity sheet (*Student Guided Practice Book*, page 23), and then have them solve Question 2 independently.

Math Fluency Games



Math Fluency Game Sets



Digital Math Fluency Games

Extend Learning

Refer students to the Lesson 3 Extend Learning Task (filename: extendtask3.pdf). Tell them that they will be finding the constant of proportionality by looking at a table. Some of the information is missing in the table. Ask students how they can determine the constant of proportionality. Students should explain that they can find the constant of proportionality by dividing any of the ordered pairs. LESSON

Math in the Real World 30 min.

- 1. Refer students to the Math in the Real World: MPG task (*Student Guided Practice Book*, page 25). Have a student read the task aloud. Tell students to explain or summarize the task to their partner. Have a few students share their summaries.
- 2. Ask students to think about what information they will need to solve the task and what the task is asking them to do. Then, have them share with a partner. Ask a few students to share out. Students need to find the constant of proportionality by dividing the dependent variable by the independent variable. They should be able to indicate that the distance depends on the number of gallons of gas in her car. Therefore, the distance is the dependent variable and the number of gallons of gas is the independent variable. Have students work in groups of two or three to complete the task.

3. As students are working, circulate and ask focusing, assessing, and advancing questions:

- What is the constant of proportionality?
- How do you use the constant of proportionality to write an equation?
- How do you know your solution is reasonable?

Sentence Frames for Explaining Reasoning

- In this problem, I needed to figure out how many _____ per _____ Meagan's car gets.
- I found the constant of proportionality by _____.
- I wrote an equation by _____
- 4. Observe how students are solving the task, and choose a few groups who solved the task in different ways to share their solutions and reasoning. Try to have the solutions move from finding the constant of proportionality to writing the equation. For example, have students share solutions with a visual representation (a graph of the information) and then the symbolic representation (equation). Students should find that the constant of proportionality is $\frac{33}{1}$ after dividing the distance by the number of gallons of gas. The equation is d = 33g. Make sure students explain their reasoning as they share solutions.

5. As groups are sharing their solution paths, reasoning, and strategies, ask questions:

- Do you agree or disagree with the solution path and reasoning? Why?
- Who can restate ____'s strategy/solution path/reasoning?
- Which solution path makes the most sense to you? Why?

Lesson Reflection (5) min.

Have students summarize their learning about how to find the constant of proportionality from tables, graphs, and equations; and provide feedback on any questions they still have about the content on the Reflection activity sheet (*Student Guided Practice Book*, page 26).

LESSON

Name:

LESSON

5

Constantly Proportional

Directions: Find the constant of proportionality for the table, graph, and equations.

1 Jill made a table for baking apple pies. Complete the table to find the constant of proportionality.

apples in one pie = apples/pie

Pies	Apples	Apples/Pie
4	20	
5	25	
6	30	
7	35	
8	40	



Constant of Proportionality:

3 What is the constant of proportionality for each equation?

$$\bigcirc \quad y = 8x$$

B
$$y = \frac{1}{2}x$$

(



Proportionally Constant

Directions: Find the constant of proportionality for the table, graph, and equations.

1 The following table represents the number of cars in the parking lots of Harris Company. Complete the table to find the constant of proportionality.

cars in one lot = cars/lot

Lots	Cars	Cars/Lot
4	24	
5	30	
6	36	
7	42	
8	48	



Constant of Proportionality:

3 What is the constant of proportionality for each equation?

$$\bigcirc \quad y = 7x$$

$$\bigcirc \quad y = \frac{1}{3}x$$

Name:

LESSON

K



Directions: Complete each of the following.

1 Find the constant of proportionality for the tables, graphs, and equations.



Date:



Refocus

Directions: Complete each of the following.

1 A local dog kennel set up a table to tell employees how many dogs can be fed given the amount of dog food. What is the constant of proportionality?

A.

Dog food (lb.)	Dogs	Dogs/Food
5	10	
10	20	
15	30	
20	40	
25	50	

Constant of Proportionality:

B. Write an equation for the table.

2 Jake made a graph of the number of party favors he wants to give to friends at his birthday party.



Name:

LESSON

Independent Practice

Directions: Find the constant of proportionality for each table and graph.

1 The following table shows the number of students in each class at Washington Middle School.

Complete the table to find the constant of proportionality.

A. students in one class = students/class

Classes	Students	Students/Class
3	21	
5	35	
7	49	
9	63	
10	70	

B. Write an equation for the table.

2 The following graph shows the number of library books as the number of students increases.



- **A.** What is the constant of proportionality of the graph?
- **B.** Write an equation for the graph.







MPG

Gas	3	5			11	13	15
Distance	99	165	264	330	363	429	



Date:____



LESSON Name: Date: K Reflection 1 How do unit rates help you make decisions about which items to purchase? 2 How does using a table help you solve problems using ratios?

Pretest

- 1. Carl keeps in shape by cycling. He bikes 9 miles every $\frac{3}{4}$ hour. How far will Carl ride in 1 hour?
 - $\bigcirc 6\frac{3}{4}$ miles
 - **B** $10\frac{1}{2}$ miles
 - Ċ 12 miles
 - **D** $14\frac{1}{2}$ miles

- **2.** Jessica runs $\frac{1}{2}$ mile in $\frac{1}{6}$ hour. How far can she run in one hour?
 - A 2 miles
 - 🕒 3 miles
 - © 4 miles
 - D 12 miles

3. The Perez family drove from their home to Washington, DC. The table shows the number of miles driven each day. Which **best** describes the relationship between the number of miles driven and the number of days?

Days	1	2	3	4
Miles	675	1,350	1,900	2,525

- (A) It is a proportional relationship because they drove the same number of miles each day.
- B It is a proportional relationship because they drove about the same number of miles each day.
- C It is not a proportional relationship because they did not drive the same number of miles each day.
- It is not a proportional relationship because they are comparing more than one quantity: miles and days.

© Teacher Created Materials

Go On

Performance Task 1: Competition Concession Stand

Part A

Belleview School is hosting the winter Belleview Band Competition. Mr. Jamison's seventh-grade class has volunteered to make the food and refreshments for the concession stand.

1. Last year, the stand sold $\frac{1}{2}$ gallon of punch in 15 minutes. Find the hourly unit rate.

- 2. Mr. Jamison's daughter is in the high school band. At a recent parade, the high school band marched $\frac{3}{4}$ of a mile in 30 minutes.
 - A How fast did they march in one hour?

B If they marched for $2\frac{1}{2}$ hours, how many miles did they march?